TD6 - Fonctions holo.

$z_1 - z_4 = -\sqrt{2}$ D'où $\mathrm{Res}(f, z_1) = \dfrac{e^{-\frac{a\sqrt{2}}{2}(1+i)}}{-2\sqrt{2}(i-1)}$

$z_1 - z_2 = -i\sqrt{2}$

$z_1 - z_3 = -\sqrt{2}(1+i)$

D'après le théorème des résidus :

$$\int_{\gamma_R} \frac{e^{iaz}}{1+z^4}\,dz = 2\pi i\left[\frac{1}{2\sqrt{2}(i-1)}\left(e^{\frac{a\sqrt{2}}{2}(i-1)} - e^{-\frac{a\sqrt{2}}{2}(1+i)}\right)\right]$$

D'où $\displaystyle\lim_{R\to\infty} \int_{\gamma_R} \frac{e^{iaz}}{1+z^4}\,dz = \int_{-\infty}^{\infty} \frac{e^{iax}}{1+x^4}\,dx$

$$= \frac{-2\pi i(1+i)}{4}\left(e^{\frac{a\sqrt{2}}{2}(i-1)} - e^{-\frac{a\sqrt{2}}{2}(1+i)}\right)$$

$$= \frac{\pi}{2}(1-i)\,e^{-\frac{a\sqrt{2}}{2}}\, 2i \sin\left(\frac{a\sqrt{2}}{2}\right) = \frac{\pi}{\cancel{4}}(1+i)\,e^{-\frac{a\sqrt{2}}{2}} \sin\left(\frac{a\sqrt{2}}{2}\right)$$

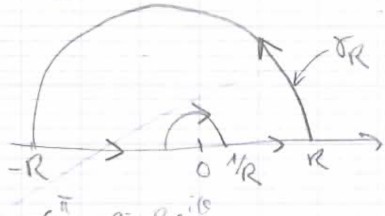
Finalement : $\displaystyle\int_{-\infty}^{\infty} \frac{e^{iax}}{1+x^2}\,dx = \pi(1+i)\,e^{-\frac{a\sqrt{2}}{2}} \sin\left(\frac{a\sqrt{2}}{2}\right)$

Et donc : $\displaystyle\int_{-\infty}^{\infty} \frac{\cos(ax)}{1+x^4}\,dx = \frac{\pi}{2}\,e^{-\frac{a\sqrt{2}}{2}} \sin\left(\frac{a\sqrt{2}}{2}\right)$

6.2.3 On pose $\displaystyle I_a = \int_0^{\infty} \frac{\cos(2ax)}{x^2}\,dx \overset{\text{parité}}{=} \frac{1}{2}\int_{-\infty}^{\infty} \frac{\cos(2ax)}{x^2}\,dx$

On remarque que $\displaystyle I_a = \frac{1}{2}\mathrm{Re}\left(\int_{-\infty}^{\infty} \frac{e^{2iax}}{x^2}\,dx\right)$

On pose : $\displaystyle J_a^R = \int_{\gamma_R} \frac{e^{2iaz}}{z^2}\,dz$



On a : $\displaystyle H_1^R = \int_0^{\pi} \frac{e^{2iaRe^{i\theta}}}{R^2 e^{2i\theta}} R i e^{i\theta}\,d\theta = \int_0^{\pi} i \frac{e^{2iaRe^{i\theta}}}{Re^{i\theta}}\,d\theta$

$\displaystyle H_2^R = \int_{-R}^{1/R} \frac{e^{2iax}}{x^2}\,dx$ / $\displaystyle H_3^R = \int_{\pi}^{0} i\, \frac{e^{2ia\frac{1}{R}e^{i\theta}}}{R^{-2}e^{+i2\theta}} \frac{1}{R} e^{+i\theta}\,d\theta$

$\displaystyle H_4^R = \int_{1/R}^{R} \frac{e^{2iax}}{x^2}\,dx$